Spring Semester 2005

Topics in Computation Theory (CS700)

Discrete Geometry

Homework 5

This homework is due on *Wednesday* **May 4**, at the beginning of the (extra) class at 4:00 p.m.

On the top of the first sheet that you turn in, please put (a) your name and student number, (b) how much time you spent working on the homework, and (c) a little table with your self-evaluation as explained on the course webpage.

1. Prove that the region *reg*(p) of a point p in the Voronoi diagram of a finite point set $P \subset \mathbb{R}^d$ is unbounded if and only if p lies on the surface of $\text{conv}(P)$.

2. (a) Let C be any circle in the plane $x_3 = 0$. Show that there exists a half-space h such that C is the vertical projection of the set $h \cap \mathcal{U}$ onto the plane $x_3 = 0$, where $\mathcal{U} = \{x \in \mathbb{R}^3 \mid x_3 = x_1^2 + x_2^2\}$ is the unit paraboloid.
   (b) Consider n arbitrary circular disks $K_1, \ldots, K_n$ in the plane. Show that the boundary of the union $\bigcup_{i=1}^n K_i$ consists of $O(n)$ circular arcs.

3. Define a "spherical polytope" as an intersection of n balls in $\mathbb{R}^3$ (such an object has facets, edges, and vertices similar to an ordinary convex polytope).
   (a) Show that any such spherical polytope in $\mathbb{R}^3$ has $O(n^2)$ faces. You may assume that the spheres are in general position.
   (b) Show that the intersection of n *unit* balls (that is, balls of radius one) has $O(n)$ complexity.

4. Prove that if $X \subset \mathbb{R}^d$ is a set where every two points have distance 1, then $|X| \leq d + 1$.

5. Give an algorithm to solve Sylvester's problem: given a set of n points in the plane, the algorithm should output a line containing exactly two points (or report that all points lie on a line). Can you achieve sub-quadratic running time? A "perfect" solution should have running time $O(n \log n)$. Bonus points will be awarded for a linear-time algorithm!