## Spring Semester 2005

## Topics in Computation Theory (CS700) Discrete Geometry Homework 4

This homework is due on *Wednesday* **April 13**, at the beginning of the (extra) class at 4:00 p.m.

On the top of the first sheet that you turn in, please put (a) your name and student number, (b) how much time you spent working on the homework, and (c) a little table with your self-evaluation as explained on the course webpage.

- 1. For a set  $X \subseteq \mathbb{R}^d$ , let  $\operatorname{ex}(X) := \{x \in X \mid x \not\in \operatorname{conv}(X \setminus \{x\})\}$  denote the set of extremal points of X. Find a convex set  $C \subseteq \mathbb{R}^d$  with  $C \neq \operatorname{conv}(\operatorname{ex}(C))$ .
- 2. Prove that if  $V \subset \mathbb{R}^d$  is affinely independent, then each subset  $F \subseteq V$  determines a face of the simplex  $\operatorname{conv}(V)$ .
- 3. Show that every convex polytope in  $\mathbb{R}^d$  is an orthogonal projection of a simplex in some higher-dimensional space  $\mathbb{R}^n$  (where we assume that  $\mathbb{R}^d$  is contained in  $\mathbb{R}^n$  as a d-dimensional linear subspace).