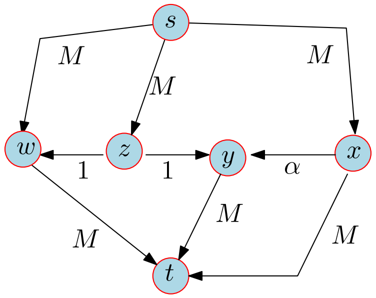
Ford-Fulkerson runs in vain



1. M: large positive integer.
2. $\alpha = (\sqrt{5} - 1)/2 \approx 0.618$.
3. $\alpha < 1$,
4. $1 - \alpha < \alpha$.
5. Maximum flow in this network is: $2M + 1$.

Some algebra...

For $\alpha = \dfrac{\sqrt{5} - 1}{2}$:

$$\begin{aligned}
\alpha^2 &= \left(\frac{\sqrt{5} - 1}{2}\right)^2 = \frac{1}{4}\left(\sqrt{5} - 1\right)^2 = \frac{1}{4}\left(5 - 2\sqrt{5} + 1\right) \\
&= 1 + \frac{1}{4}\left(2 - 2\sqrt{5}\right) \\
&= 1 + \frac{1}{2}\left(1 - \sqrt{5}\right) \\
&= 1 - \frac{\sqrt{5} - 1}{2} \\
&= 1 - \alpha.
\end{aligned}$$

Some algebra...

Claim

Given: $\alpha = (\sqrt{5} - 1)/2$ *and* $\alpha^2 = 1 - \alpha$.

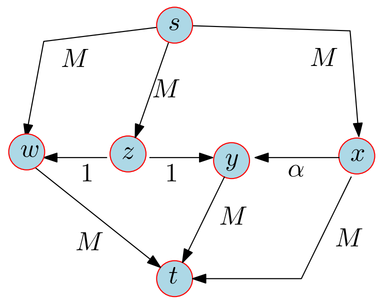
$$\implies \forall i \qquad \alpha^i - \alpha^{i+1} = \alpha^{i+2}$$

Proof.

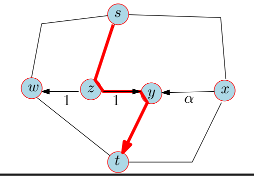
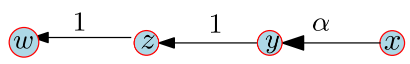
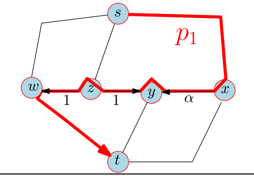
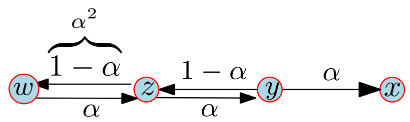
$$\alpha^i - \alpha^{i+1} = \alpha^i(1 - \alpha) = \alpha^i \alpha^2 = \alpha^{i+2}.$$

□

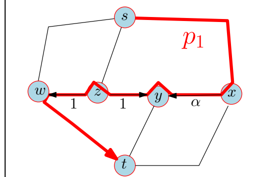
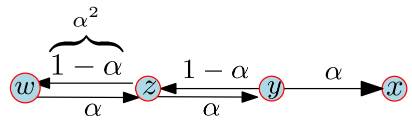
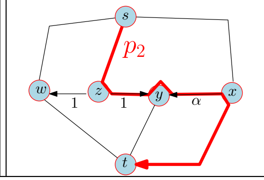
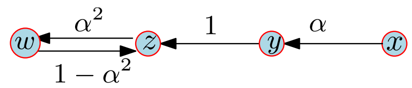
The network



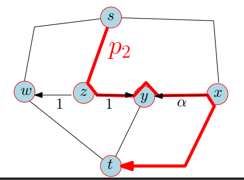
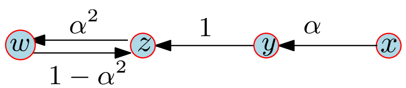
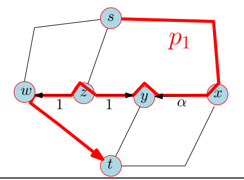
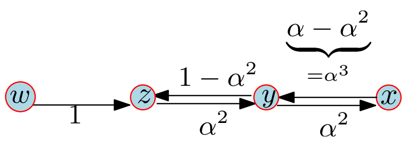
Let it flow...

#	Augment. path π	c_π	New residual network
0.		**1**	
1.		α	

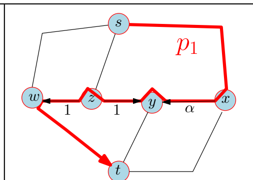
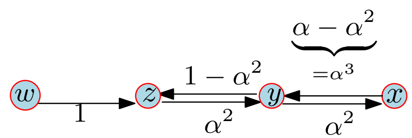
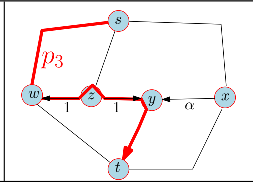
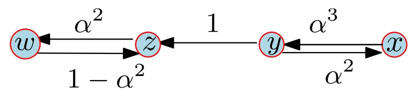
Let it flow II

#	Augment. path π	c_π	New residual network
1.		α	
2.		α	

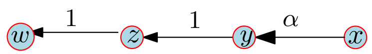
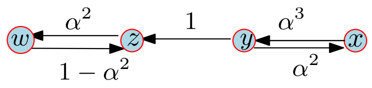
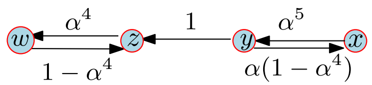
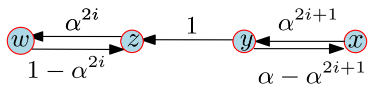
Let it flow II

2.		α^2	
3.		α^2	

Let it flow III

3.		α^2	
4.		α^2	

Let it flow III

moves	Residual network after
0	
moves **0, (1, 2, 3, 4)**	
moves **0, (1, 2, 3, 4)^2**	
0.(1, 2, 3, 4)^i	

Namely, the algorithm never terminates.