CS 500

April 8, 2015

PARTITION $\leq$ LOAD BALANCING

$X = \{x_1, x_2 \dots x_n\} \longrightarrow n, m, t_1 \dots t_n$

Is there subset S $\qquad t_i = x_i \quad m = 2$

s.t $\underbrace{\sum_{i \in S} x_i = \sum_{i \notin S} x_i}$

$$\sum_{i \in S} x_i = \frac{\sum x_i}{2} \iff \max\left(\sum_{i \in S} x_i, \sum_{i \notin S} x_i\right) = \frac{\sum x_i}{2}$$

1, 2, 5	7	
6, 3	6	7
4	6	

$\sum t_i = 19$

3 machines

$$\underbrace{Opt(X) \leq Greedy(X) \leq 2\, Opt(X)}$$

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① $T^* \geq \max_j t_j$ ② $T^* \geq \frac{1}{m} \sum_j t_j$

Consider greedy schedule. $T_i = Greedy(X)$

$T_i \geq T_k \quad k \in \{1 \dots m\}$

$T_i - t_j \leq T_k \leq T_i$

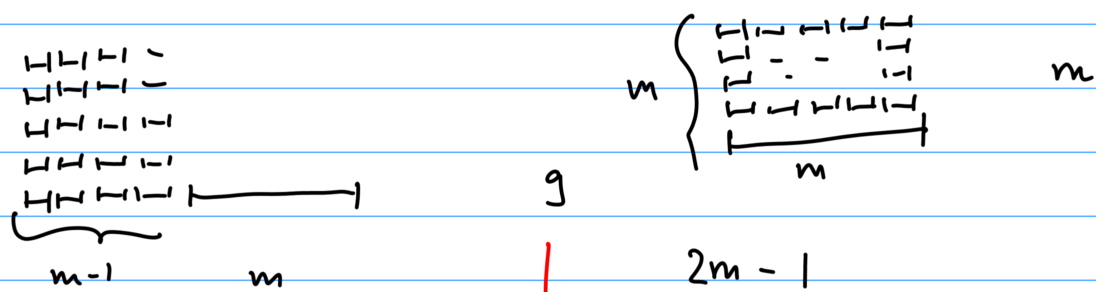
$$\sum_k T_k \geq \sum_k (T_i - t_j) = m(T_i - t_j)$$

$$T_i \leq \frac{1}{m}\underbrace{\left(\sum_k T_k\right)}_{\sum_j t_j} + \underbrace{t_j}_{\leq \max t_j}$$

$$T_i \leq \frac{1}{m}\sum_j t_j + \max t_j \leq T^* + T^* = 2T^* \qquad \square$$

$m(m-1)$ short jobs $t_i = 1$

1 long job $t_n = m$



$$\overbrace{t_1 \geq t_2 \geq t_3 \geq \dots \geq t_m \geq t_{m+1}} \geq \dots \geq t_n$$

m+1 jobs

$T^* \geq 2t_{m+1}$

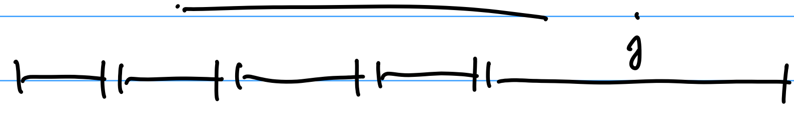
$t_i \geq t_{m+1}$

$t_{m+1} = \min\{t_i \mid 1 \leq i \leq m+1\}$

Some machine has at least 2 jobs of length $\geq t_{m+1}$

$\Rightarrow T^* \geq 2t_{m+1}$ $\square$

i

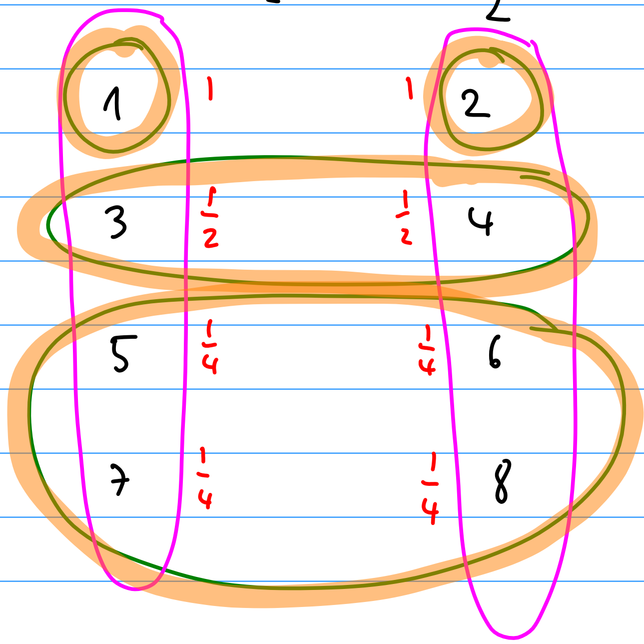


$j \geq m+1$

$t_j \leq t_{m+1} \leq \frac{1}{2}T^*$

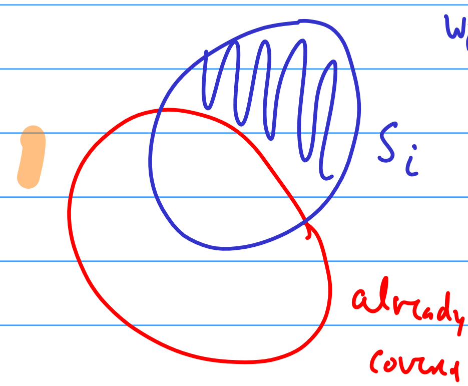
$$T_i \leq \frac{1}{m}\sum_j t_j + t_j$$

$$T_i \le T^* + \frac{1}{2}T^* = \frac{3}{2}T^*$$



= weight 1

= weight 1.0001



GReedy $S^1 \; S^2 \; \dots \; S^k$ is a cover $\bigcup_{1\le i\le k} S^i = U$

$s \in U$ $\quad S^i$ is first to cover s

$$c_s = \frac{w(S^i)}{|S^i \cap R|}$$

charging argument

$$H(n) = \sum_{i=1}^{n} \frac{1}{i} \approx \ln n \qquad n\text{th Harmonic number}$$

$$\boxed{\sum_{s \in S_k} c_s \leq H(|S_k|) \cdot w_k}$$

$S_k = \{ s_1 \cdots \underbrace{s_j \; s_d}_{\subseteq R} \}$ in order of being covered.

$d - j + 1$

$$c_{s_j} \leq w_k / d-j+1$$

$$\sum_{s \in S_K} c_s \leq w_k \sum_{j=1}^{d} \frac{1}{d-j+1} = w_k \left(1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{d} \right)$$

$$= H(d) \cdot w_k$$

$\mathcal{C}$ = greedy cover $\qquad \mathcal{C}^*$ = optimal cover

$$w_i \geq \frac{1}{H(d^*)} \sum_{s \in S_i} c_s$$

$$OPT = w(C^*) = w^* = \sum_{i \in \mathcal{C}^*} w_i \geq \sum_{i \in C^*} \frac{1}{H(d^*)} \sum_{s \in S_i} c_s$$

$$\boxed{H(d^*) \cdot w^*} \geq \sum_{i \in \mathcal{C}^*} \sum_{s \in S_i} c_s \geq \sum_{s \in U} c_s = w(\mathcal{C})$$

greedy cover weight

$d^* \leq n \qquad \ln n$ - approx