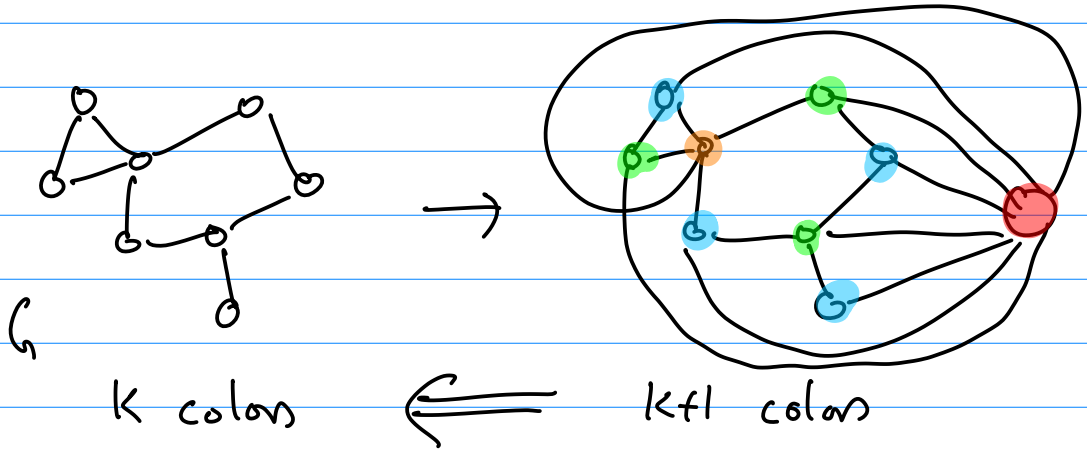
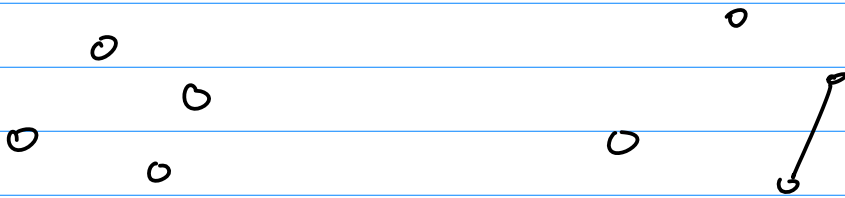


G is k colorable $\iff G'$ is $(k+1)$ -colorable

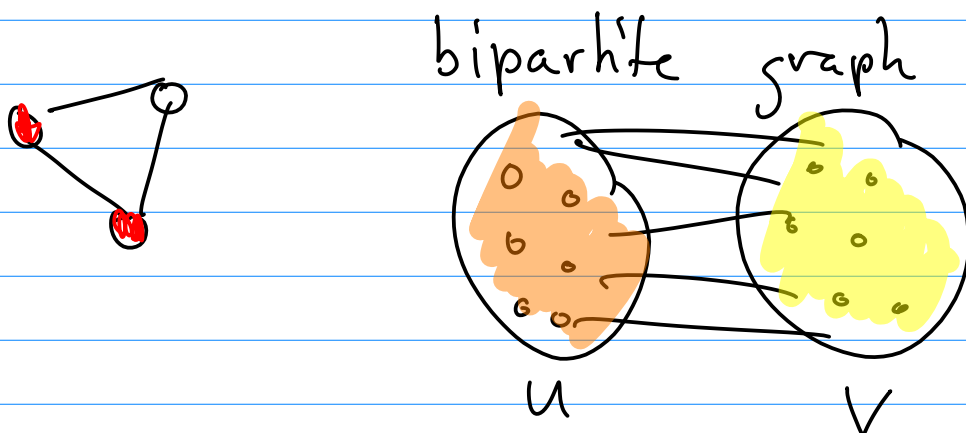


G' $(k+1)$ colorable $\implies G$ is k -colorable

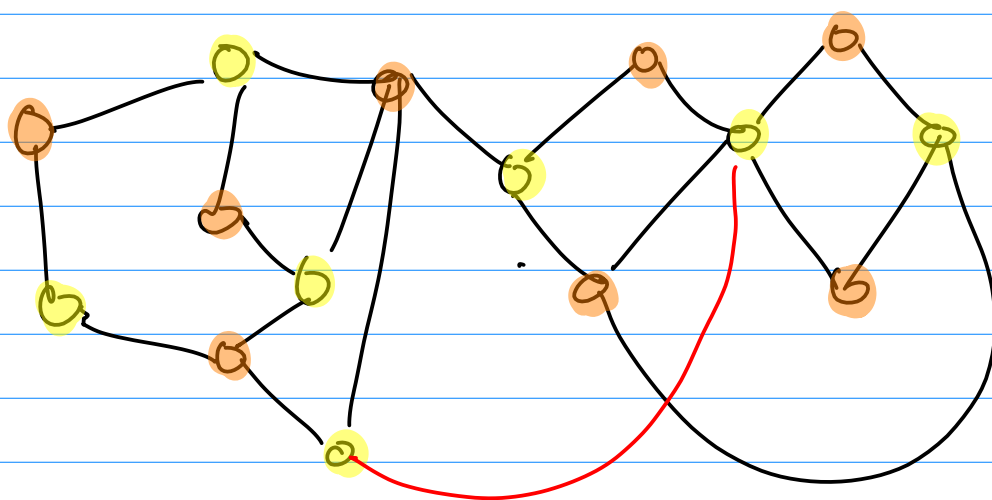
Reduction is poly-time $O(|V|)$



2-coloring is linear time



G 2-colorable $\Leftrightarrow G$ is bipartite



DFS

BFS

$O(|V| + |E|)$

2-COLORING

\leq

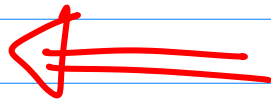
3-COLORING

linear time

nobody knows

G $\xrightarrow{\text{poly-time}}$ (G', k)

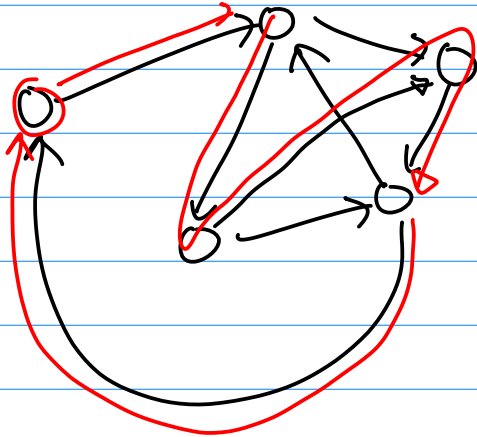
G is 3-colorable $\iff G'$ has ind. set S of size $\geq k$



G' has ind. set S of size $\geq n$

- ① $|S| = n$ because S contains ≤ 1 vtx from each triangle
- ② S contains one vertex from each triangle
- ③ \implies can define colors for vertices of G
- ④ This is a 3-coloring of G

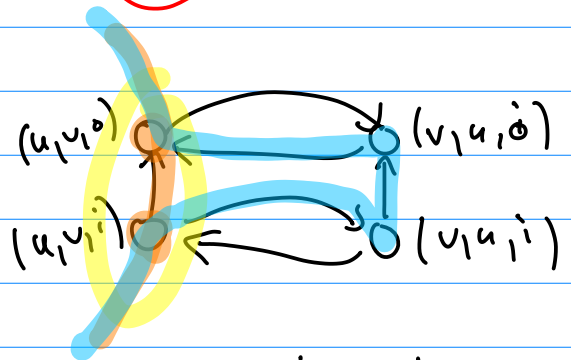
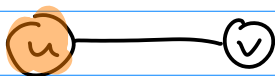
$2\text{-COLORING} \geq \text{Ind Set} \geq 3\text{-COLORING} \leq 4\text{-COLORING}$
 $\text{Ind Set} \geq \text{VERTEX COVER}$



$(G, k) \xrightarrow{\text{poly-time}} G'$
 directed graph

G has V.C. of size $\leq k \iff G'$ has H.C.

m edges $\longrightarrow 4m + k$ vertices



n vertex chains