Homework – CS492 – due April 11, 2008

Problem 1. Give examples of free $\mathbb{Z}_2$-spaces of index n that are not $(n-1)$-connected.

Problem 2. Give an example of a free $\mathbb{Z}_2$-space (X,α) with $\text{ind}_{\mathbb{Z}_2}(X,\alpha) = \infty$.

Problem 3. Let

$$V_{n,2} = \{(v_1, v_2) \in (S^{n-1})^2 \ : \ v_1 \cdot v_2 = 0\} \subset \mathbb{R}^{2n}$$

be the Steifel manifold of pairs of orthogonal unit vectors, $n \geq 1$ (with the topology of a subspace of $\mathbb{R}^{2n}$). Let α be the $\mathbb{Z}_2$-action given by $(v_1, v_2) \mapsto (-v_1, -v_2)$.

a. Show that $\text{ind}_{\mathbb{Z}_2}(V_{n,2}, \alpha) \leq n-1$.

b. Describe a $\mathbb{Z}_2$-map $S^1 \to V_{2,2}$, proving that $\text{ind}_{\mathbb{Z}_2}(V_{2,2}, \alpha) = 1$.

c. Describe a $\mathbb{Z}_2$-map $S^1 \to V_{3,2}$.

d. Describe a $\mathbb{Z}_2$-map $S^3 \to V_{4,2}$, proving that $\text{ind}_{\mathbb{Z}_2}(V_{4,2}, \alpha) = 3$.

Problem 4. Let (X,α) be a free $\mathbb{Z}_2$-space, and let $A, B \subset X$ be closed invariant sets (that is, $\alpha(A) = A$ and $\alpha(B) = B$) with $A \cup B = X$. Show that

$$\text{ind}_{\mathbb{Z}_2}(X,\alpha) \leq \text{ind}_{\mathbb{Z}_2}(A,\alpha) + \text{ind}_{\mathbb{Z}_2}(B,\alpha) + 1$$